# Mini-Lecture 9.1

The Logic in Constructing Confidence Intervals for a Population Mean Where the Population Standard Deviation Is Known

**Objectives**

1. Compute a point estimate of the population mean
2. Construct and interpret a confidence interval about the population mean, assuming the population standard deviation is known
3. Understand the role of margin of error in constructing a confidence interval
4. Determine the sample size necessary for estimating the population mean within a specified margin of error

**Examples**

1. The Graduate Management Admission Test (GMAT) is a test required for admission into many masters of business administration (MBA) programs. Total scores on the GMAT are normally distributed and historically have a population standard deviation of 113. The Graduate Management Admission Council (GMAC), who administers the test, claims that the mean total score is 529. (Source: www.mba.com/NR/rdonlyres/55DF55BA-4F4E-4DB9-A5BE-39DC98C46551/0/ExamineeScoreGuide5.pdf.) Suppose a random sample of 8 students took the test, and their scores are given below.

   699, 560, 414, 570, 521, 663, 727, 413

   a. Find a point estimate of the population mean. (570.9)
   b. Construct a 95% confidence interval for the true mean score for the population. (492.6, 649.2)
   c. Does this interval contain the value reported by GMAC? (Yes)
   d. How many students should be surveyed to estimate the mean score within 25 points with 90% confidence? (56 students)
   e. How many students should be surveyed to estimate the mean score within 25 points with 95% confidence? (79 students)
   f. How many students should be surveyed to estimate the mean score within 25 points with 99% confidence? (136 students)

2. The Graduate Record Examination (GRE) is a test required for admission to many U.S. graduate schools. Students' scores on the verbal reasoning portion of the GRE follow a normal distribution with a population standard deviation of 117. The Educational Testing Service (ETS), which administers the exam, claims that the mean verbal reasoning score is 465. (Source: http://www.ets.org/Media/Tests/GRE/pdf/994994.pdf) Suppose a random sample of 10 students took the test, and their scores are given below.

   489, 564, 624, 284, 388, 424, 515, 361, 398, 546

   a. Find a point estimate of the population mean. (459.3)
   b. Construct a 95% confidence interval for the true mean score for this population. (386.8, 531.8)
   c. Does this interval contain the mean score claimed by ETS? (Yes)
   d. How many students should be surveyed to estimate the mean score within 50 points with 95% confidence? (21 students)
   e. How many students should be surveyed to estimate the mean score within 25 points with 95% confidence? (85 students)
   f. How many students should be surveyed to estimate the mean score within 5 points with 95% confidence? (2104 students)

**In-class Activity**

(Requires one six-sided die for each student) Have each student toss a dice n=10 times and compute the mean of their observations. Given that the population standard deviation is approximately 1.707, have the students create their own 90% confidence intervals for the true mean. Remind the students that the true mean is 3.5 and ask how many of them had confidence intervals that contained this value. (Note: With n=10 observations from this discrete uniform distribution, the distribution of the sample mean is approximately normal.)

# Mini-Lecture 9.2

Confidence Intervals for a Population Mean in Practice Where the Population Standard Deviation Is Unknown

**Objectives**

1. Know the properties of Student's *t*-distribution
2. Determine *t*-values
3. Construct and interpret a confidence interval for a population mean

**Examples**

1. A group of Brigham Young University—Idaho students (Matthew Herring, Nathan Spencer, Mark Walker, and Mark Steiner) collected data in November 2005 on the speed of vehicles traveling through a construction zone on a state highway, where the posted speed was 25 mph. The recorded speed (in mph) of 14 randomly selected vehicles is given below.
   20, 24, 27, 28, 29, 30, 32, 33, 34, 36, 38, 39, 40, 40
   a. Assuming speeds are approximately normally distributed, construct a 95% confidence interval for the true mean speed of drivers in this construction zone. Interpret the interval. ((28.6, 35.7); we are 95% confident that the true mean speed of drivers is between 28.6 and 35.7 mph. *Note: this does not mean that 95% of all drivers travel between 28.6 and 35.7 mph on this road; it also does not imply that the probability that a randomly selected driver's speed will be between 28.6 and 35.7 is 95%*.)
   b. Construct a 99% confidence interval for the true mean speed of drivers in this construction zone. Interpret the interval. ((27.2, 37.1); we are 99% confident that the true mean speed of drivers is between 27.2 and 37.1 mph.)
   c. Compare the widths of the 95% and 99% confidence intervals. (The 95% confidence interval has width 7.1, and the 99% confidence interval has width 9.9, so the 99% confidence interval is wider.)
   d. What conclusions do you draw about the speeds people drive in this construction zone? (There is reason to believe that the mean speed of drivers in this area exceeds the posted speed limit.)

2. The heights of adult men are normally distributed. The heights (in inches) of 9 young adult males are given below.
   72, 71, 71, 68, 68.75, 70.25, 72, 70.25, 68.25
   a. Construct a 95% confidence interval for the true mean height (in inches) of young adult men. Interpret the interval. ((69.0, 71.3); we are 95% confident that the true mean height of young adult males is between 69.0 and 71.3 inches.)
   b. Construct a 90% confidence interval for the true mean height (in inches) of young adult men. Interpret the interval. ((69.2, 71.1); we are 90% confident that the true mean height of young adult males is between 69.2 and 71.1 inches.)

**In-class Activity**

Ask a random sample of students to state their height in inches. Choose students who all have the same gender. (Note: Some students may need to be reminded how to calculate their height in inches. For example, a 5-foot 10½-inch person would report a height of 5(12)+10.5=70.5 inches.) Assuming heights are approximately normally distributed, construct a 95% confidence interval for the true mean height of adults of the chosen gender.

# Mini-Lecture 9.3

Confidence Intervals for a Population Proportion

**Objectives**

1. Obtain a point estimate for the population proportion
2. Construct and interpret a confidence interval for the population proportion
3. Determine the sample size necessary for estimating a population proportion within a specified margin of error

**Examples**

1. As a potential worldwide pandemic, avian influenza H5N1 (commonly called the bird flu) poses a serious health risk. As of April 30, 2008, there have been 382 human cases of this virus in the world. Of these cases, 241 have resulted in death. (Source: http://www.who.int/csr/disease/avian_influenza/en/) Consider the outcomes of these cases as a random sample of all possible outcomes.
    a. Find a point estimate for the proportion of people who would die if infected with the bird flu. (0.631)
    b. Construct a 90% confidence interval for the proportion of cases that would be expected to result in death if a pandemic occurred. (0.590, 0.672)
    c. Interpret the confidence interval. (We are 90% confident that the true proportion of cases that result in death is between 59.0% and 67.2%)

2. Nitrates are groundwater contaminants derived from fertilizer, septic tank seepage, and other sewage. Nitrate poisoning is particularly hazardous to infants under the age of 6 months. The Maximum Contaminant Level (MCL) is the highest level of a contaminant the government allows in drinking water. For nitrates, the MCL is 10 mg/L. (Source: Environmental Protection Agency. "Ground Water and Drinking Water: Current Drinking Water Standards." 15 Feb 2008. www.epa.gov/safewater/mcl.html.) The health department wants to know what proportion of wells in Madison County that have nitrate levels above the MCL. A worker has been assigned to take a simple random sample of wells in the county, measure the nitrate levels, and assess compliance. What size sample should the health department obtain if the estimate is desired to be within 2 percent with 95% confidence if
    a. there is no prior information available? (2401 wells)
    b. a study conducted two years ago showed that approximately 7% of the wells in Madison County had nitrate levels exceeding the MCL? (626 wells)

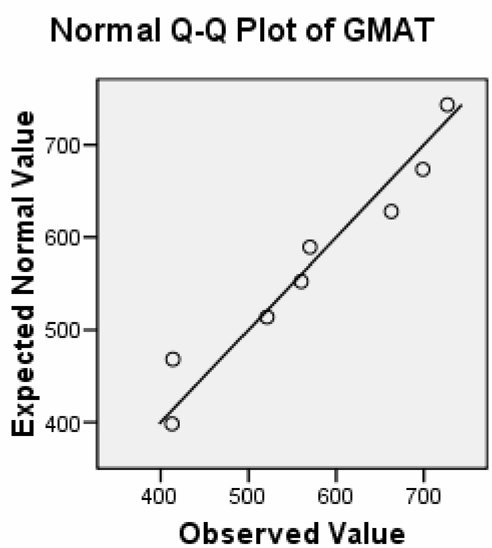
# Mini-Lecture 9.4
Confidence Intervals for a Population Standard Deviation

**Objectives**
1. Find critical values for the chi-square distribution
2. Construct and interpret confidence intervals for the population variance and standard deviation

**Examples**
1. The Graduate Management Admission Test (GMAT) is a test required for admission into many masters of business administration (MBA) programs. Total scores on the GMAT are normally distributed and historically have a mean of 529 and a standard deviation of 113. (Source: http://www.mba.com/NR/rdonlyres/55DF55BA-4F4E-4DB9-A5BE-39DC98C46551/0/ExamineeScoreGuide5.pdf.) Suppose a random sample of 8 students took the test, and their scores are given below.

   699, 560, 414, 570, 521, 663, 727, 413

   a. Verify that the data are normally distributed by constructing a normal probability plot.

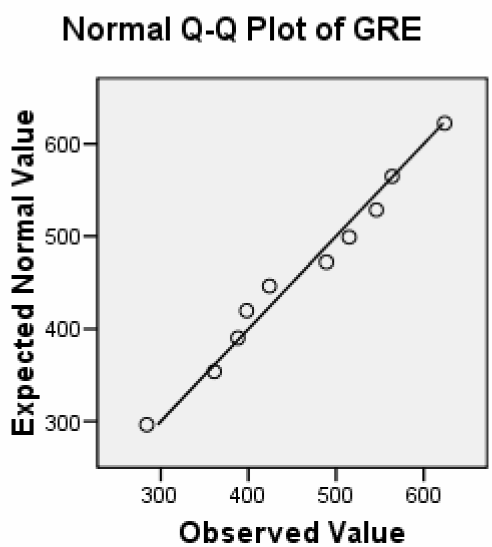



   b. Determine the sample standard deviation. ($s = 120.4$)
   c. Construct a 95% confidence interval for the population standard deviation. (79.6, 245.0)
   d. Does this interval substantiate GMAC's claim? (Yes)

2. The Graduate Record Examination (GRE) is a test required for admission to many U.S. graduate schools. The Educational Testing Service (ETS), which produces the test, claims that students' scores on the verbal reasoning portion of the GRE are normally distributed with a mean of 465 and a standard deviation of 117. (Source: http://www.ets.org/Media/Tests/GRE/pdf/994994.pdf) Suppose a random sample of 10 students took the test, and their scores are given below.

   489, 564, 624, 284, 388, 424, 515, 361, 398, 546

   a. Verify that the data are normally distributed by constructing a normal probability plot.




   b. Determine the sample standard deviation. ($s = 105.4$)
   c. Construct a 95% confidence interval for the population standard deviation. (72.5, 192.4)
   d. Does this interval substantiate the ETS's claim? (Yes)

# Mini-Lecture 9.5

Putting It Together: Which Procedure Do I Use?

**Objective**

1. Determine the appropriate confidence interval to construct

**Examples**

1. After World War II, under the rule of the Communist party, opponents of the Romanian government were imprisoned and tortured. A sample of 59 political detainees in Romania were examined and it was determined that 32 suffered from lifetime posttraumatic stress disorder (PTSD). (Source: Bichescu D, et al. (2005) Long-term consequences of traumatic experiences: an assessment of former political detainees in Romania. Clinical Practice and Epidemiology in Mental Health (1)17.) What type of confidence interval should be constructed? Construct a 95% confidence interval for the prevalence of PTSD among Romanian political detainees. (Confidence interval for a population proportion; (0.415, 0.669))

2. The heights of 15 randomly selected adult women were measured. The heights (in inches) are given below.
   71, 66.75, 65, 66.5, 65.75, 65.25, 67.5, 65.75, 68.5, 68.75, 67, 67.75, 64.5, 67.75, 66
   Assuming heights are approximately normally distributed.
   a. Suppose you want to estimate the true mean height of women in the population. What type of confidence interval should be constructed? Construct the appropriate 95% confidence interval. (Confidence interval for a population mean, where the standard deviation is unknown; (66.0, 67.9))
   b. Suppose you want to estimate the spread of the height of women in the population (in inches). What type of confidence interval should be constructed? Construct the appropriate 95% confidence interval. (Confidence interval for a population standard deviation; (1.24, 2.68))