Mini-Lecture 8.1
Distribution of the Sample Mean

Objectives
1. Describe the distribution of the sample mean - samples from normal populations
2. Describe the distribution of the sample mean - samples from a population that is not normal

Examples
1. The combined (verbal + quantitative reasoning) score on the GRE is normally distributed with mean 1049 and standard deviation 189. (Source: http://www.ets.org/Media/Tests/GRE/pdf/994994.pdf.) Suppose \( n = 16 \) randomly selected students take the GRE on the same day.
   a. What is the probability that one randomly selected student scores above 1100 on the GRE? (0.3936)
   b. Describe the sampling distribution of the sample mean for the 16 students. (It is normal with mean 1049 and standard deviation 47.25)
   c. What is the probability that a random sample of 16 students has a mean GRE score that is less than 1100? (0.8599 [Tech: 8598])
   d. What is the probability that a random sample of 16 students has a mean GRE score that is 1100 or above? (0.1401 [Tech: 0.1402])

2. In the United States, the year each coin was minted is printed on the coin. To find the age of a coin, simply subtract the current year from the year printed on the coin. The ages of circulating pennies are right skewed. Most circulating pennies were minted relatively recently, and extremely old pennies are rare. Assume the ages of circulating pennies have a mean of 12.2 years and a standard deviation of 9.9 years.
   a. Based on the information given, can we determine the probability that a randomly selected penny is over 10 years old? (No, because the population of the ages of circulating pennies is not normally distributed.)
   b. What is the probability that a random sample of 40 circulating pennies has a mean less than 10 years? (0.0793 [Tech: 0.0799])
   c. What is the probability that a random sample of 40 circulating pennies has a mean greater than 10 years? (0.9207 [Tech: 0.9201])
   d. What is the probability that a random sample of 40 circulating pennies has a mean greater than 15 years? Would this be unusual? (0.0367 [Tech: 0.0368]; yes)
Mini-Lecture 8.2
Distribution of the Sample Proportion

Objectives
1. Describe the sampling distribution of a sample proportion
2. Compute probabilities of a sample proportion

Examples
1. According to a recent article, 98% of the people who travel on the New Jersey Turnpike exceed the 60 miles per hour speed limit. (Source: Lange JE, Johnson MB, Voas RB (2005) Testing the Racial Profiling Hypothesis for Seemingly Disparate Traffic Stops. Justice Quarterly (22)2.) Law enforcement officials are planning to covertly measure the speed of 1,000 cars on the Turnpike and will compute the proportion of drivers who exceed the posted speed limit.
   a. Describe the sampling distribution of \( \hat{p} \), the sample proportion of drivers who exceed the speed limit. (Approximately normal with mean 0.98 and standard deviation 0.0044)
   b. In a random sample of 1000 drivers, what is the probability that more than 97% of drivers will be speeding? (0.9881)
   c. Would it be unusual if in a random sample of 1000 drivers, 97% were speeding? (Yes, since the probability that 97% or less would be speeding is 0.0119.)
   d. What is the minimum number of drivers that must be sampled to be sure that \( \hat{p} \) is approximately normal? (511)

2. The quality control director of a small company that manufactures exercise equipment knows that 10% of their products are defective. Market research shows that only 30% of their customers will use the product in the first year after the sale. So, the manufacturer expects that 3% of the units will be returned for service under a one-year full warranty. This company expects to sell 600 units next year. (This problem is based on data from an actual company.)
   a. Describe the sampling distribution of \( \hat{p} \), the proportion of the 600 units that will be returned for service. (Approximately normal with mean 0.03 and standard deviation 0.006964)
   b. What is the probability that more than 3% of the 600 units will be returned for service? (0.5)
   c. What is the probability that more than 5% of the 600 units will be returned for service? (0.0021 [Tech: 0.0020])
   d. Would it be unusual if in a random sample of 600 units, 5% were returned for service? (Yes)