Mini-Lecture 7.1

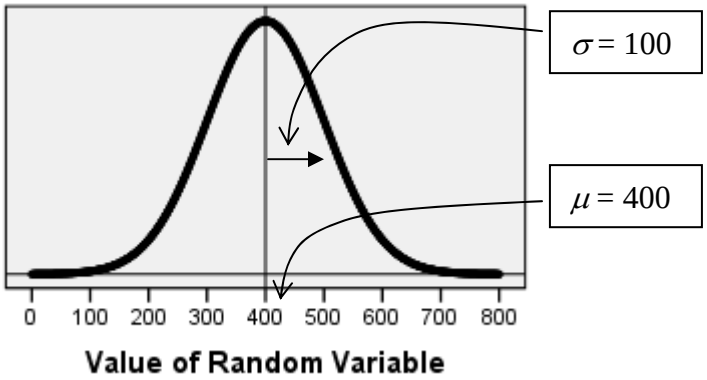
Properties of the Normal Distribution

Objectives

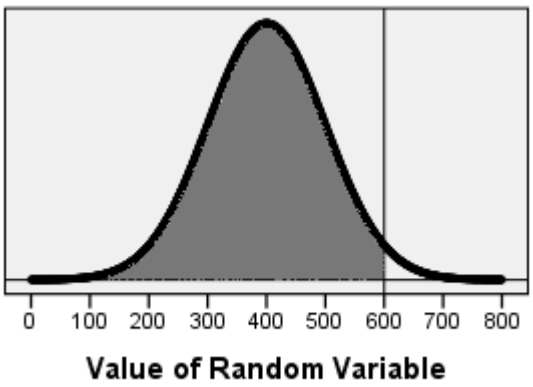
1. Understand the uniform probability distribution
2. Graph a normal curve
3. State the properties of the normal curve
4. Understand the role of area in the normal density function
5. Understand the relation between a normal random variable and a standard normal random variable

Examples

1. Suppose a random variable is normally distributed with a mean of 400 and a standard deviation of 100.
   a. Draw a normal curve with the parameters labeled. (Solution below.)

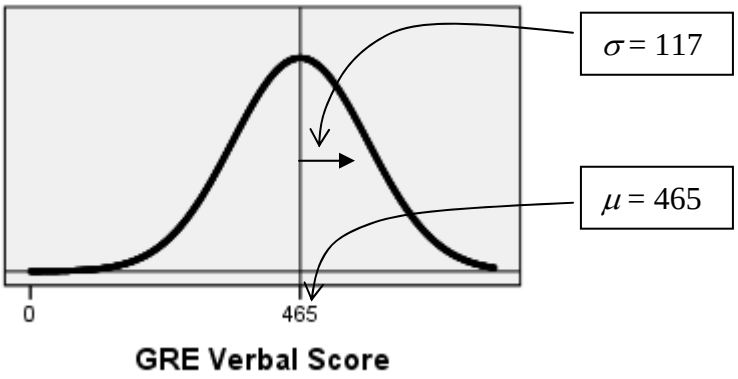


   b. Shade the region under the normal curve that represents that probability of observing a value of the random variable that is less than 600. (Solution below.)

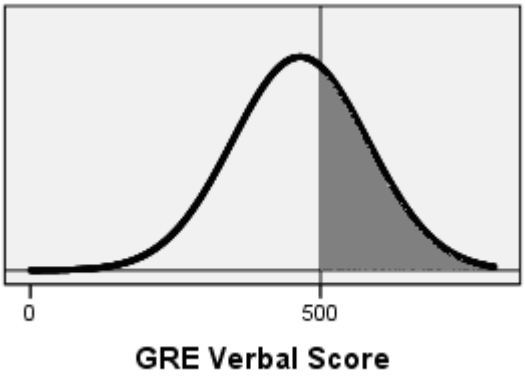


   c. Suppose the area under the normal curve below 600 is 0.9772. Provide two interpretations of this area. ((i) The proportion of observations of the random variable that will be below 600 is 0.9772, (ii) the probability that a value of this random variable will be below 600 is 0.9772.)

2. The Graduate Record Examination (GRE) is a test required for admission to many U.S. graduate schools. Students' scores on the verbal reasoning portion of the GRE follow a normal distribution with mean 465 and standard deviation 117. (Source: http://www.ets.org/Media/Tests/GRE/pdf/994994.pdf.) In addition to other qualifications, a score of at least 500 on the verbal section of the GRE required for admission to the master's program at the School of Library and Information Science at Indiana University (Source: http://www.slis.indiana.edu/admissions/admission_criteria.php.)
   a. Draw a normal curve with the parameters labeled. (Solution below.)



   b. Shade the region under the normal curve that represents test takers who scored above 500 on the verbal portion of the GRE. (Solution below.)



   c. Suppose the area under the normal curve above 500 is 0.4880. Provide two interpretations of this area. ((i) The proportion of students who score above 500 is 0.3824, (ii) the probability that a randomly selected student will score above 500 is 0.3824.)

Mini-Lecture 7.2

The Standard Normal Distribution

Objectives

1. Find the area under the standard normal curve
2. Find *Z*-scores for a given area
3. Interpret the area under the standard normal curve as a probability

Examples

1. Determine the area under the standard normal curve that lies
   a. to the left of $Z = -2.31$. (0.0104)
   b. to the right of $Z = -1.47$. (0.9292)
   c. between $Z = -2.31$ and $Z = 0$. (0.4896)
   d. between $Z = -2.31$ and $Z = -1.47$. (0.0603)
   e. between $Z = 1.47$ and $Z = 2.31$. (0.0603)
   f. between $Z = -2.31$ and $Z = 1.47$. (0.9188)
   g. to the left of $Z = -2.31$ or to the right of $Z = 1.47$. (0.0812)

2. The Graduate Record Examination (GRE) is a test required for admission to many U.S. graduate schools. The Department of Health Policy & Management in the Graduate School of Public Health at the University of Pittsburgh requires a score on the verbal portion of the GRE that is at least the 60^{th} percentile. (Source: http://www.publichealth.pitt.edu/interior.php?pageID=101.)
   a. Find the *Z*-score corresponding to the 60^{th} percentile. In other words, find the *Z*-score such that the area under the standard normal curve to the left is 0.60. (0.25)
   b. How many standard deviations above the mean is the 60^{th} percentile? (0.25)

Mini-Lecture 7.3

Applications of the Normal Distribution

Objectives

1. Find and interpret the area under a normal curve
2. Find the value of a normal random variable

Examples

1. The combined (verbal + quantitative reasoning) score on the GRE is normally distributed with mean 1049 and standard deviation 189. (Source: http://www.ets.org/Media/Tests/GRE/pdf/994994.pdf.) The Department of Psychology at Columbia University in New York requires a minimum combined score of 1200 for admission to their doctoral program. (Source: www.columbia.edu/cu/gsas/departments/psychology/department.html.)
    a. What proportion of combined GRE scores can be expected to be under 1100? (0.6064)
    b. What proportion of combined GRE scores can be expected to be over 1100? (0.3936)
    c. What proportion of combined GRE scores can be expected to be between 950 and 1000? (0.0959 [Tech: 0.0975])
    d. What is the probability that a randomly selected student will score over 1200 points? (0.2119 [Tech: 0.2122])
    e. What is the probability that a randomly selected student will score less than 1049 points? (0.5000)
    f. What is the percentile rank of a student who earns a combined GRE score of 1300? (91^{st} percentile)
    g. What is the percentile rank of a student who earns a combined GRE score of 1000? (40^{th} percentile)
    h. Determine the 70^{th} percentile of combined GRE scores. (1148)
    i. Determine the combined GRE scores that make up the middle 95% of all scores. (679 to 1419)
    j. Compare the results in part (i) to the values given by the Empirical Rule. (671 to 1427; they are very close, since the Empirical Rule is based on the normal distribution)

2. The diameters of ball bearings produced at a factory are approximately normally distributed. Suppose the mean diameter is 1.002 centimeters (cm) and the standard deviation is 0.006 cm. The product specifications require that the diameter of each ball bearing be between 0.980 and 1.020 cm.
    a. What proportion of ball bearings can be expected to have a diameter under 1.020 cm? (0.9987)
    b. What proportion of ball bearings can be expected to have a diameter over 1.020 cm? (0.0013)
    c. What proportion of ball bearings can be expected to have a diameter between 0.980 and 1.020 cm? That is, what proportion of ball bearings can be expected to meet the specifications? (0.9986)

    d. What is the probability that the diameter of a randomly selected ball bearing will be over 1.000 cm? (0.6292 [Tech: 0.6306])
    e. What is the probability that the diameter of a randomly selected ball bearing will be under 0.995 cm? (0.1210 [Tech: 0.1217])
    f. What is the percentile rank of a ball bearing that has a diameter of 0.991 cm? (3rd percentile)
    g. What is the percentile rank of a ball bearing that has a diameter of 1.011 cm? (93rd percentile)
    h. Determine the 10th percentile of the diameters of ball bearings. (0.994 cm)
    i. Determine the diameters of ball bearings that make up the middle 99% of all diameters. (0.987 to 1.017 cm)

Mini-Lecture 7.4

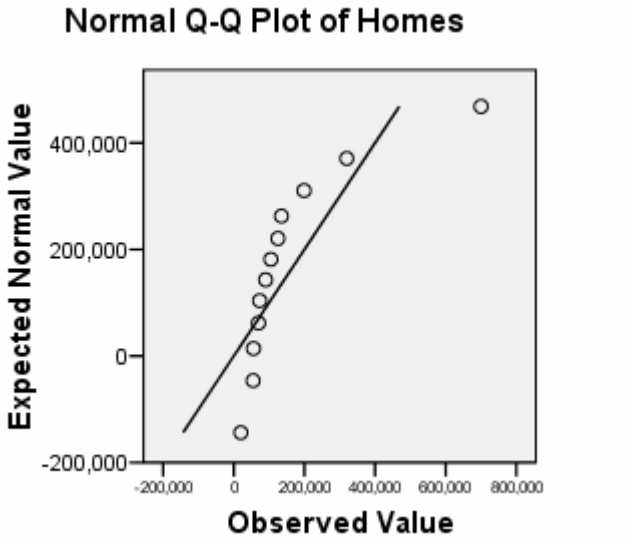
Assessing Normality

Objective

1. Draw normal probability plots to assess normality

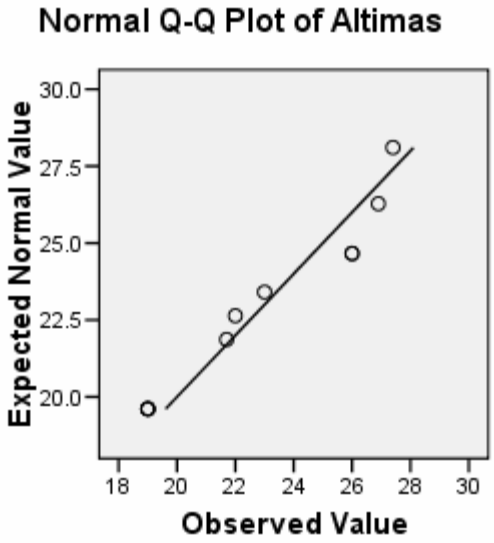
Examples

1. Twelve single-family homes for sale in May 2008 in Springfield, Illinois were randomly selected. The listed prices for these homes are given below. (Source: http://www.realtor.com.) Use a normal probability plot to assess whether the sample data could have come from a population that is normally distributed.
   104900, 55900, 69900, 198900, 319900, 19900, 54900, 73500, 90000, 125000, 699900, 135000



   (The data do not appear to follow a normal distribution.)

2. The website cars.com gives the selling price of many vehicles listed for sale in a given locality. The advertised prices (in thousands) of eleven one-year-old Nissan Altima cars listed for sale in the Boston area are given below. (Source: www.cars.com.) Use a normal probability plot to assess whether the sample data could have come from a population that is normally distributed.
   26, 19, 19, 19, 26.9, 27.4, 23, 26, 22, 21.7, 19

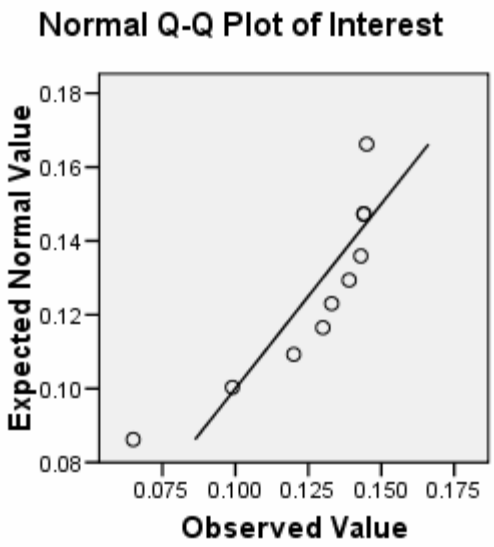


(Answers may vary; the data appear to follow a normal distribution.)

3. Every six months, the United States Federal Reserve Board conducts a survey of credit card plans in the U.S. The following data are the interest rates charged by 10 credit card issuers randomly selected for the January 2008 survey. (Source: http://www.federalreserve.gov/pubs/SHOP/survey.htm)

Institution	**Rate**
Pulaski Bank and Trust Company	6.5%
Rainier Pacific Savings Bank	12.0%
Wells Fargo Bank NA	14.4%
Firstbank of Colorado	14.4%
Lafayette Ambassador Bank	14.3%
Infibank	13.0%
United Bank, Inc.	13.3%
First National Bank of The Mid-Cities	13.9%
Bank of Louisiana	9.9%
Bar Harbor Bank and Trust Company	14.5%

Use a normal probability plot to assess whether the sample data could have come from a population that is normally distributed.

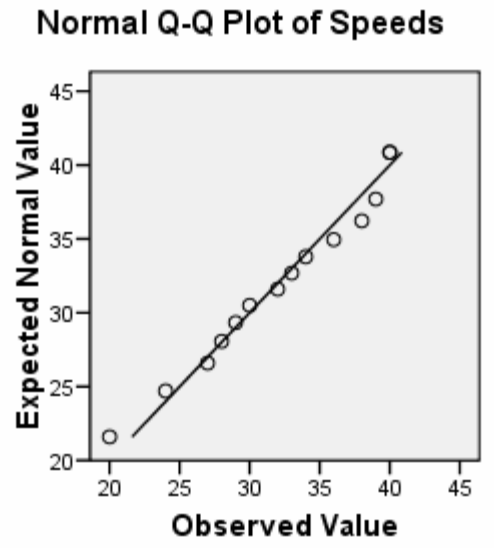


(The data do not appear to be normally distributed.)

4. A group of Brigham Young University—Idaho students (Matthew Herring, Nathan Spencer, Mark Walker, and Mark Steiner) collected data in November 2005 on the speed of vehicles traveling through a construction zone on a state highway, where the posted speed was 25 mph. The recorded speed of 14 randomly selected vehicles is given below:

   20, 24, 27, 28, 29, 30, 32, 33, 34, 36, 38, 39, 40, 40

   Create a normal probability plot to assess the normality of these data.



(The data appear to be normally distributed. Compare this to the conclusion drawn from the histogram in Section 3.1.)

Mini-Lecture 7.5

The Normal Approximation to the Binomial Probability Distribution

Objectives

1. Approximate binomial probabilities using the normal distribution

Examples

1. In a poll conducted in April 2008, 27% of residents of the United States are renting the home in which they live. (Source: http://surveys.ap.org) Suppose 200 people are randomly selected. Use the normal approximation to the binomial to
    a. approximate the probability that exactly 54 of them will report renting their home. (0.0638 [Tech: 0.0635])
    b. approximate the probability that more than 45 of them will report renting their home. (0.9115 [Tech: 0.9121])
    c. approximate the probability that more than 60 of them will report renting their home. (0.1492 [Tech: 0.1503])
    d. approximate the probability that the number of respondents who will report renting their home is between 46 and 60, inclusive. (0.7623 [Tech: 0.7618])

2. Darren did not prepare for a multiple-choice test in his biochemistry class. He is forced to randomly guess on each question. Suppose the test has 60 questions with 4 possible answers for each question. Use the normal approximation to the binomial to
    a. approximate the probability that he will get exactly 12 questions right. (0.0774 [Tech: 0.0797])
    b. approximate the probability that he will get more than 15 questions right. (0.4404 [Tech: 0.4407])
    c. approximate the probability that he will get at least 21 questions right. (0.0505)
    d. approximate the probability that he will get more than 25 questions right. (0.0009)
    e. approximate the probability that the number of questions he will get is between 12 and 17, inclusive. (0.6242 [Tech: 0.6236])

3. Most nurseries guarantee their plants for one year. If the plant dies within the first year of the purchase date, it can be returned for credit. Experience has shown that about 5% of all plants purchased from Evergreen Nursery will die in their first year. Evergreen Nursery sold 250 walnut trees this season. Use the normal approximation to the binomial to
    a. approximate the probability that exactly 12 walnut trees will be returned for credit. (0.1141 [Tech: 0.1142])
    b. approximate the probability that 15 or fewer walnut trees will be returned for credit. (0.8078 [Tech: 0.8080])
    c. approximate the probability that more than 20 walnut trees will be returned for credit. (0.0102 [Tech: 0.0101])

d. approximate the probability that more than 25 walnut trees will be returned for credit. (0.0001)

In-class Activity

(Requires one coin for each student) Have each student flip a coin n=55 times, counting the number of heads. (The odd number of flips prevents students from having exactly 50% heads.) Have each student do the following.

a. Check that the assumptions to use the normal approximation to the binomial are satisfied.
b. Use the normal approximation to the binomial to approximate the probability that they would get exactly as many heads as they observed, assuming the coin is fair.
c. Use the normal approximation to the binomial to approximate the probability that they would get at least as many heads as they observed, assuming the coin is fair.
d. Use the normal approximation to the binomial to approximate probability that they would get more heads than they observed, assuming the coin is fair.