Mini-Lecture 6.1
Discrete Random Variables

Objectives
1. Distinguish between discrete and continuous random variables
2. Identify discrete probability distributions
3. Construct probability histograms
4. Compute and interpret the mean of a discrete random variable
5. Interpret the mean of a discrete random variable as an expected value
6. Compute the variance and standard deviation of a discrete random variable

Examples
1. A group of students plan to purchase a 16 ounce bag of M&M’s and record data on the contents. For each of the following variables, state whether it is discrete or continuous.
   a. Mass of each of the M&M’s (continuous)
   b. Number of pieces of candy in the bag (discrete)
   c. Diameter of each piece of candy (continuous)
   d. Number of distinct colors of M&M’s in the bag (discrete)


   At a monthly ‘casino night,’ there is a game called Chuck-a-Luck: Three dice are rolled in a wire cage. You place a bet on any number from 1 to 6. If any one of the three dice comes up with your number, you win the amount of your bet. (You also get your original stake back.) If more than one die comes up with your number, you win the amount of your bet for each match. For example, if you had a $1 bet on number 5, and each of the dice came up with 5, you would win $3. It appears that the odds of winning are 1 in 6 for each of the three dice, for a total of 3 out of 6 - or 50%. Adding the possibility of having more than one die come up with your number, the odds would seem to be in the gambler's favor. What are the odds of winning this game? I can't believe that a casino game would favor the gambler.

   Daniel computed the probabilities incorrectly. There are four possible outcomes. (The selected number can match 0, 1, 2, or 3 of the dice.) The random variable \( X \) represents the profit from a $1 bet in Chuck-A-Luck. The table below summarizes the probabilities of earning a profit of \( x \) dollars from a $1 bet. Use this table to answer the following questions.

<table>
<thead>
<tr>
<th>Number of dice matching the chosen number</th>
<th>Profit $x$</th>
<th>Probability $P(X=x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 dice</td>
<td>$-1$</td>
<td>125 / 216</td>
</tr>
<tr>
<td>1 dice</td>
<td>$1$</td>
<td>75 / 216</td>
</tr>
<tr>
<td>2 dice</td>
<td>$2$</td>
<td>15 / 216</td>
</tr>
<tr>
<td>3 dice</td>
<td>$3$</td>
<td>1 / 216</td>
</tr>
</tbody>
</table>

a. Verify that this is a discrete probability distribution. (The probabilities are all between 0 and 1, and they sum to 1)

b. Draw a probability histogram for the profit.

![Probability Histogram for Profit](image)

c. Compute and interpret the mean of the random variable $X$. ($-0.08$. If you play this game for a very long time, you will lose approximately $0.08 per game.)

d. Based on your answer to the previous question, would you recommend playing this game? (No)

e. Compute the variance of the random variable $X$. (1.239)

f. Compute the standard deviation of the random variable $X$. (1.113)

g. What is the probability that a player who plays once will lose this game? In other words, what is the probability that a player will match none of the three dice? (125/216)

h. What is the probability that a player will match all three of the dice?
i. What is the probability that a player will match at least one of the dice?
Mini-Lecture 6.2
The Binomial Probability Distribution

Objectives
1. Determine whether a probability experiment is a binomial experiment
2. Compute probabilities of binomial experiments
3. Compute the mean and standard deviation of a binomial random variable
4. Construct binomial probability histograms

Examples
1. After World War II, under the rule of the Communist party, some opponents of the Romanian government were imprisoned and tortured. A random sample of 59 former political detainees in Romania (none of whom know each other) was examined and the number of former detainees who suffered from lifetime posttraumatic stress disorder (PTSD) was determined. Answer the following questions. (Source: Bichescu D, et al. (2005) Long-term consequences of traumatic experiences: an assessment of former political detainees in Romania. Clinical Practice and Epidemiology in Mental Health (1)17.)
   a. Explain why this is a binomial experiment. (There are a fixed number of trials (presence of PTSD in the 59 former detainees), the trials are independent (the incidence of PTSD is independent for each subject), there are only two outcomes (presence or absence of PTSD), the probability of developing PTSD is the same for each former detainee, and we are interested in the number of subjects who suffer from PTSD.)
   b. Suppose that half of the former political detainees in Romania suffer from PTSD, what is the mean of \( X \), the number of people suffering from PTSD in a sample of 59 former detainees? (29.5)
   c. Interpret the mean. (Assuming that half of the former detainees suffer from PTSD, we expect that in a sample of size 59, we expect that 29.5 of the subjects will suffer from PTSD.)
   d. Compute the standard deviation of \( X \). (3.84)
   e. Would it be unusual to find 32 individuals suffering from PTSD in a group of 59 former political detainees? (No, since this is less than one standard deviation above the mean.)
2. To avoid unpleasant surprises when the statement comes, you try to record all your credit card transactions in a ledger. Unfortunately, you tend to neglect recording about 5% of your purchases. Suppose that last month, you had 25 purchases on your credit card account. When the statement arrives, you count the number of purchases you forgot to record. The random variable $X$ represents the number of unrecorded purchases in a month with 25 transactions.

   a. Explain why this is a binomial experiment. (There is a fixed number of trials (25), the trials are independent (whether we record a transaction or not does not depend on recording another transaction), there are only two outcomes (recorded or not), the probability is the same for each trial (0.05), and we are interested in the number of transactions that are not recorded.)

   b. Find and interpret the mean of $X$. (1.25; this is the expected number of transactions that would be unrecorded)

   c. Compute the standard deviation of $X$. (1.09)

   d. Find the probability that you would record all 25 purchases. (0.2774)

   e. Find the probability that exactly 4 purchases would have been unrecorded. (0.0269)

   f. Find the probability that fewer than 4 purchases would have been unrecorded. (0.9659)

   g. Find the probability that at least 4 purchases would have been unrecorded. (0.0341)

   h. Would it be unusual to find 4 unrecorded purchases in a month with 25 purchases? (Yes)
Mini-Lecture 6.3
The Poisson Probability Distribution

Objectives
1. Understand when a probability experiment follows a Poisson process
2. Compute probabilities of a Poisson random variable
3. Find the mean and standard deviation of a Poisson random variable

Examples
1. According to the Atlantic Oceanographic and Meteorological Laboratory, an average of 10.0 tropical storms occurred per year between 1951 and 2000. (Source: www.aoml.noaa.gov/hrd/Landsea/deadly/Table6.htm). Compute the probability that the number of tropical storms next year will be
   a. exactly zero. Interpret the result. (0.00005; it is very unlikely there will be zero tropical storms next year)
   b. exactly eight tropical storms. Interpret the result. (0.1126; it is not unlikely that there will be exactly eight tropical storms next year)
   c. exactly ten tropical storms. Interpret the result. (0.1251; it is not unlikely that there will be zero tropical storms next year)
   d. exactly 25 tropical storms, as in the 2005 season. Interpret the result. (Source: www.weather.com/newscenter/tropical) (0.00003; it is very unlikely that there will be exactly 25 tropical storms)

2. Based on your cell phone records, you notice that you receive an average of 6 telephone calls per day. Find that probability that tomorrow you will receive
   a. no telephone calls. Interpret the result. (0.0025; it is unlikely that you will receive no telephone calls tomorrow.)
   b. exactly 1 call. Interpret the result. (0.0149; it is unlikely that you will receive exactly one telephone call tomorrow.)
   c. exactly 6 calls. Interpret the result. (0.1606; it is not unlikely that you will receive exactly 6 telephone calls tomorrow.)
   d. at least 4 calls. Interpret the result. (0.8488; it is very likely that you will receive at least 4 telephone calls tomorrow.)
Mini-Lecture 6.4 (on CD)
The Hypergeometric Probability Distribution

Objectives
1. Determine whether a probability experiment is a hypergeometric experiment
2. Compute the probabilities of hypergeometric experiments
3. Compute the mean and standard deviation of a hypergeometric random variable

Examples
1. In the multi-state lottery game Powerball, players choose five distinct integers between 1 and 55. Twice a week, a drawing is held. For the drawing, 55 numbered white balls are placed in a bin. Five of these balls are drawn at random without replacement, and the corresponding numbers are recorded. Players win money based on how many of the numbers they guessed correctly. (Source: www.Powerball.com)
   a. Let the random variable $X$ represent how many of the five numbers were guessed correctly. Does this represent a hypergeometric probability experiment? (Yes)
   b. Find the probability that a randomly selected player will correctly guess the values for none of the five white balls. (0.6091)
   c. Find the probability that a randomly selected player will correctly guess the values for exactly one of the five white balls. (0.3310)
   d. Find the probability that a randomly selected player will correctly guess the values for exactly two of the five white balls. (0.0563)
   e. Find the probability that a randomly selected player will correctly guess the values for exactly three of the five white balls. (0.0035)
   f. Find the probability that a randomly selected player will correctly guess the values for all five of the white balls. (2.87 x $10^{-7}$)
   g. Find the mean and standard deviation of the number of white balls that will be correctly guessed. (0.4545; 0.6186)

Note to instructor: The following problems require information from previous sections of the textbook.

h. Find the probability that a randomly selected player will correctly guess at least one of the white balls. Note: this does not mean the player won any money, it just means that they guessed at least one of the white balls correctly. (0.3909)

i. It can be shown that that the overall probability of winning money (even a small amount) by playing Powerball is 0.0273. In other words, over 97% of the time, people lose when playing Powerball. Even though people lose almost every time they play Powerball, how do you think the probability you calculated in Part (h) might psychologically impact the people who play Powerball? (Since the probability of getting at least one match is very high, people might be encouraged to continue playing.)
2. When playing the Powerball lottery, in addition to guessing the value of five white balls, players also guess the “Powerball.” At the time of the semi-weekly drawings, 42 red balls are placed in a bin. One of these is selected as the Powerball. (Source: www.Powerball.com)
   a. Let the random variable \( Y \) be 1 if a player correctly guesses the Powerball and 0 if they do not. Note that this represents the number of Powerballs that were correctly guessed, where the maximum value is 1. Does this represent a hypergeometric probability experiment? \( \text{(Yes)} \)
   b. Use the hypergeometric probability distribution to find the probability that a randomly selected player will correctly guess the Powerball. \( \text{(0.0238)} \)

Note to instructor: The following problems require information from previous sections of the textbook.

c. Repeat Part (b) using classical probability calculations. \( \frac{1}{42} = 0.0238 \)

d. Random variables \( X \) and \( Y \) are independent. Find the probability that a randomly selected player will correctly guess the values for exactly three of the five white balls and will fail to correctly guess the Powerball. \( \text{(0.0034)} \)

e. Random variables \( X \) and \( Y \) are independent. Find the probability that a randomly selected player will correctly guess the values all five white balls and the Powerball. This is the probability that a randomly selected player will win the jackpot. \( \text{(6.84 \times 10^{-9})} \)